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COMMENT

Comment on 'New relations between the Clebsch–Gordan coefficients of SU(2)'

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Abstract. The relations investigated by Kulesza and Rembieliński are particular cases of a recoupling equation.

Kulesza and Rembieliński (1980) obtained relations between Clebsch–Gordan coefficients of the type (their equation (12))

$$\sum_{sm} \omega^{s}_{[AB],[A'B']}\langle AB\, ab|sm\rangle\langle A'B'\, a'b'|sm\rangle = 0.$$

Relations of this kind can be obtained from the recoupling equation (which follows, for example, from equation (2.20) of Rotenberg *et al* (1959))

$$\sum_{sm} (-1)^{A+B+s}\begin{Bmatrix} s & A & B \\ \lambda & B' & A' \end{Bmatrix}\langle AB\, ab|sm\rangle\langle A'B'\, a'b'|sm\rangle = \frac{(-1)^{A-a+B'-b'}}{2\lambda+1}\langle AA'\, a-a'|\lambda\, a-a'\rangle\langle BB'\, b-b'|\lambda\, b-b'\rangle \qquad \text{(R)}$$

by setting $|a-a'|>\lambda$ so that the right-hand side vanishes.

Equation (24) of Kulesza and Rembieliński (1980) is obtained from the recoupling equation (R) when λ takes the smallest meaningful value $n_0 = \max(|A-A'|, |B-B'|)$:

$$\omega^{s}_{[AB],[A'B']_{\min}} = G(-1)^{A+B+s}\begin{Bmatrix} s & A & B \\ n_0 & B' & A' \end{Bmatrix}$$

where

$$G = \left(\frac{(2A+1)!(A+B-A'-B')!(A-B-A'+B')!(A+B-A'+B'+1)!}{(2A-2A')!(2A')!(-A+B+A'+B')!}\right)^{1/2}$$

(in the case $n_0 = A - A'$) as results from equation (22.14) of Jucys and Bandzaitis (1965) for the stretched $6j$ coefficient.

Equations (26) and (27) of Kulesza and Rembieliński (1980) correspond to

$$\omega^{s}_{[AB][A'B']} = G[s(s+1)]^n(-1)^{A+B+s}\begin{Bmatrix} s & A & B \\ n_0 & B' & A' \end{Bmatrix},$$

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which, using the recurrence relations (23.17) of Jucys and Bandzaitis (1965) is seen to be of the form

$$\omega^{s}_{[AB][A'B']} = \sum_{k=0}^{n} C_k (-1)^{A+B+s} \begin{Bmatrix} s & A & B \\ n_0+k & B' & A' \end{Bmatrix}$$

where the C_k do not depend on s. Equation (26) or (27) is then the linear combination of equations (R) for $\lambda = n_0 + k$ with coefficients C_k $(0 \leqslant k \leqslant n)$.

References

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